# A Tactile Sensing for Estimating the Position and Orientation of a Joint-Axis of a Linked Object 

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#### Abstract

This paper describes a tactile sensing to estimate the position and orientation of a joint-axis of a linked object. This tactile sensing is useful when a multi-jointed multi-fingered robotic hand manipulates a tool which has a joint. This estimation requires sensing of the location of a contact point and the direction of an edge of the tool as contact information measured by a robotic fingertip. A conventional hard fingertip with a force sensor can measure only the location of a contact point. In contrast, we have already developed a robotic fingertip with a force sensor and a soft skin, and it can measure not only the location of a contact point but also the direction of an edge of an object. The estimation of a joint-axis of a linked object is demonstrated by using the soft fingertip.


## I. Introduction

A multi-jointed multi-fingered robotic hand has potential capability of dexterous manipulation [1], [2]. A human hand demonstrates high dexterity by handling various tools. If the robotic hand can use a tool, it improves its dexterity. However, it is difficult for the robotic hand to use a tool. Because a measurement of the relative position and orientation between a target object, a tool, and the hand is required for handling a tool. So few work is reported on the robotic hand handling a tool [3]. If a tool has a joint, tool handling is more difficult. Because the robotic hand system has to additionally consider the relative position and orientation between the direction of a joint-axis of the tool and the target object. For example, when the robotic hand cuts a paper with scissors, blades are perpendicular to the paper. In other words, the direction of the joint-axis of scissors is kept to be parallel to the paper (Fig. 1). Mutual occlusion will often occur because a target object, a tool, and the robotic hand are very close to each other while the hand uses the tool. Then, a vision system will easily fail to measure their position and orientation. So, we propose a tactile sensing of estimating the position and orientation of a joint-axis of a tool.

Katz et al. proposed a method of estimating the position of a joint-axis of a tool through point features tracking using a vision sensor [4]. In this method, a vision system needs to continuously track feature points on the surface of a tool while a robotic hand uses the tool. However, the vision system will often fail to track feature points due to occlusion, lighting condition, and complex background during a tool handling. In contrast to this, a tactile sensing can avoid these failure factors of the vision sensing because a tactile sensor directly contacts a target. So, we try to replace a visual feature tracking by a tactile sensing of the loci of

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Fig. 1. Direction of the joint-axis of scissors.
contact points (Fig. 2). Katz's method estimates the position of a joint-axis of a tool on the assumption that visual point features do not move on the surface of the tool. However, the contact point often moves on the surface of the object due to slip and rolling during a tool handling process. Therefore, error of the estimation will increase if contact points are used instead of visual point features.


Fig. 2. Tactile sensing for estimating the position and orientation of a joint-axis of a linked object.


Fig. 3. Contact point moves on an edge of a link of an object.
We propose another approach using a geometrical constraint between the joint-axis of the tool and an edge of a link of the tool. A distance between the joint-axis and the extended line of an edge is always constant during a tool handling process. We assume that the contact point moves on an edge (Fig. 3). Under these constraint conditions, we can estimate the position and orientation of the jointaxis from tactile information. A measurement of the tactile
information including the location of a contact point and the direction of an edge is required for the implementation of this approach. A conventional hard robotic fingertip with a six-axis force/torque sensor can measure only the location of a contact point. In contrast, we have already proposed a method of measuring the direction of an edge in addition to the location of a contact point by using a robotic fingertip with a six-axis force/torque sensor and a soft skin [5], [6].

In this paper, we propose a tactile sensing of estimating the position and orientation of a joint-axis of a tool. The method is robust against not only the failure factors with the vision sensing but also the movement of a contact point on an edge.

## II. Estimation Method of Position and Orientation of Joint-Axis

We measure the location of a contact point and the direction of an edge of the object as contact information. The contact information is measured by a robotic fingertip equipped with a 6 -axis force/torque sensor and a soft skin. The contact information is measured several times during a rotation of the joint of the object. Then, the position and orientation of the joint-axis is estimated from the contact information. We assume that the object has one revolute joint and two links.

The distance between the edge of the object and the jointaxis of the object is constant during a rotation of the revolute joint. Therefore, we can estimate the position and orientation of the joint-axis of the object by using the measured contact information (Fig. 4). The contact information is measured $N$ times during the rotation of the joint. The edge of the object is expressed as:

$$
\begin{equation*}
(x y z)^{T}=\boldsymbol{x}_{\boldsymbol{i}}+t \boldsymbol{e}_{\boldsymbol{i}} \tag{1}
\end{equation*}
$$

where $\boldsymbol{x}_{\boldsymbol{i}}=\left(x_{i} y_{i} z_{i}\right)^{T}$ is the location of the contact point at the $i$ th measurement, $\boldsymbol{e}_{\boldsymbol{i}}=\left(e_{x i} e_{y i} e_{z i}\right)^{T},\left(\left|\boldsymbol{e}_{\boldsymbol{i}}\right|=1\right)$ is the direction of the edge of the object at the $i$ th measurement, and $t$ is a scalar parameter. Moreover, the joint-axis of the object is expressed as:

$$
\begin{align*}
(x y z)^{T} & =(p q r)^{T}+s(u v w)^{T} \\
& =\boldsymbol{p}+s \boldsymbol{u}  \tag{2}\\
u^{2}+v^{2}+w^{2} & =1 \tag{3}
\end{align*}
$$

where $\boldsymbol{p}=(p q r)^{T}$ is the position of a point on the jointaxis, $\boldsymbol{u}=(u v w)^{T},(|\boldsymbol{u}|=1)$ is the orientation of the jointaxis, and $s$ is a scalar parameter. The minimum of $\left|\boldsymbol{a}_{\boldsymbol{i}}-\boldsymbol{a}\right|$ is the distance between the edge and the joint-axis, where $\boldsymbol{a}_{\boldsymbol{i}}=\left(a_{i} b_{i} c_{i}\right)^{T}$ is a point on the $i$ th edge (Eq. (1)) and $\boldsymbol{a}=(a b c)^{T}$ is a point on the joint-axis (Eq. (2)). When $\left|\boldsymbol{a}_{\boldsymbol{i}}-\boldsymbol{a}\right|$ is the minimum value, $\left(\boldsymbol{a}_{\boldsymbol{i}}-\boldsymbol{a}\right) \perp \boldsymbol{e}_{\boldsymbol{i}}$ and $\left(\boldsymbol{a}_{i}-\boldsymbol{a}\right) \perp$ $\boldsymbol{u}$ are valid.

$$
\left\{\begin{array}{l}
\left(\boldsymbol{a}_{\boldsymbol{i}}-\boldsymbol{a}\right) \cdot \boldsymbol{e}_{\boldsymbol{i}}=0  \tag{4}\\
\left(\boldsymbol{a}_{\boldsymbol{i}}-\boldsymbol{a}\right) \cdot \boldsymbol{u}=0
\end{array}\right.
$$

To find the solution to these equations, substitute the values: $\boldsymbol{a}_{\boldsymbol{i}}=\boldsymbol{x}_{\boldsymbol{i}}+t \boldsymbol{e}_{\boldsymbol{i}}$ and $\boldsymbol{a}=\boldsymbol{p}+s \boldsymbol{u}$.

$$
\left\{\begin{align*}
\left(x_{i}-p+t \boldsymbol{e}_{i}-s u\right) \cdot \boldsymbol{e}_{\boldsymbol{i}} & =0  \tag{5}\\
\left(\boldsymbol{x}_{i}-\boldsymbol{p}+t \boldsymbol{e}_{\boldsymbol{i}}-s u\right) \cdot \boldsymbol{u} & =0
\end{align*}\right.
$$

By solving simultaneous equations (5),

$$
\begin{align*}
s & =\frac{\left|\boldsymbol{e}_{\boldsymbol{i}}\right|^{2}\left(\boldsymbol{x}_{\boldsymbol{i}}-\boldsymbol{p}\right) \cdot \boldsymbol{u}-\left(\boldsymbol{e}_{\boldsymbol{i}} \cdot \boldsymbol{u}\right)\left(\boldsymbol{x}_{\boldsymbol{i}}-\boldsymbol{p}\right) \cdot \boldsymbol{e}_{\boldsymbol{i}}}{\left|\boldsymbol{e}_{\boldsymbol{i}}\right|^{2}|\boldsymbol{u}|^{2}-\left(\boldsymbol{e}_{\boldsymbol{i}} \cdot \boldsymbol{u}\right)}  \tag{6}\\
t & =\frac{\left(\boldsymbol{e}_{\boldsymbol{i}} \cdot \boldsymbol{u}\right)\left(\boldsymbol{x}_{\boldsymbol{i}}-\boldsymbol{p}\right) \cdot \boldsymbol{u}-|\boldsymbol{u}|^{2}\left(\boldsymbol{x}_{\boldsymbol{i}}-\boldsymbol{p}\right) \cdot \boldsymbol{e}_{\boldsymbol{i}}}{\left|\boldsymbol{e}_{\boldsymbol{i}}\right|^{2}|\boldsymbol{u}|^{2}-\left(\boldsymbol{e}_{\boldsymbol{i}} \cdot \boldsymbol{u}\right)} \tag{7}
\end{align*}
$$

are obtained. The distance between the edge and the jointaxis is expressed as:

$$
\begin{equation*}
\left|\boldsymbol{x}_{\boldsymbol{i}}-\boldsymbol{p}+t_{d} \boldsymbol{e}_{\boldsymbol{i}}-s_{d} \boldsymbol{u}\right|=d \tag{8}
\end{equation*}
$$

where $d$ is the distance between the edge and the jointaxis, $s_{d}$ and $t_{d}$ are $s$ at equation (6) and $t$ at equation (7), respectively. Seven parameters $\left(\boldsymbol{p}=(p q r)^{T}, \boldsymbol{u}=(u v w)^{T}\right.$, and $d$ ) are estimated by using equation (3) and equations (8) obtained through six tactile measurements.

The denominators of $s_{d}$ and $t_{d}$ are equal to zero when the edge and the joint-axis are parallel to each other ( $\boldsymbol{e}_{\boldsymbol{i}}=\boldsymbol{u}$ ). When $\boldsymbol{e}_{\boldsymbol{i}}$ is equal to $\boldsymbol{u}$,

$$
\begin{equation*}
s-t=\frac{\left(\boldsymbol{x}_{\boldsymbol{i}}-\boldsymbol{p}\right) \cdot \boldsymbol{e}_{\boldsymbol{i}}}{\left|\boldsymbol{e}_{\boldsymbol{i}}\right|^{2}} \tag{9}
\end{equation*}
$$

is obtained by solving simultaneous equations (5). Then, the position and orientation of the joint-axis is estimated from equations (8) by using $s_{d}^{\prime}$ and $t_{d}^{\prime}$. These $s_{d}^{\prime}$ and $t_{d}^{\prime}$ are expressed as:

$$
\begin{equation*}
s_{d}^{\prime}=\frac{\left(\boldsymbol{x}_{\boldsymbol{i}}-\boldsymbol{p}\right) \cdot \boldsymbol{e}_{\boldsymbol{i}}}{\left|\boldsymbol{e}_{\boldsymbol{i}}\right|^{2}}, t_{d}^{\prime}=0 \tag{10}
\end{equation*}
$$



Fig. 4. Defined vectors to calculate the position and orientation of the jointaxis: $\boldsymbol{x}_{\boldsymbol{i}}$ is the position vector of the contact point at the $i$ th measurement, $\boldsymbol{e}_{\boldsymbol{i}}$ is the direction vector of the edge of the object at the $i$ th measurement, $\boldsymbol{p}_{\boldsymbol{i}}$ is the position vector of a point on the joint-axis, $\boldsymbol{u}$ is the orientation of the joint-axis, and $d=\left|a_{i}-a\right|$ is the distance between the edge and the joint-axis.

## III. Measurement of Contact Information

We formalize a method of measuring both the location of a contact point and the direction of an edge of an object by using a robotic fingertip with a six-axis force/torque sensor and a soft skin.

## A. Contact Model

In the case of contact with deep penetration of an object into the soft fingertip, the geometric property of the object affects the transmissible friction moment at the contact area. We formalize a contact model between a soft fingertip and a stick-like object. This contact model is an extension to Mason and Salisbury's contact model [7]. When the soft fingertip comes into contact with a stick-like object that includes, for example, a sharp wedge, a rounded wedge, or the edge of a plate, the shape of the contact area on the fingertip becomes heteromorphic. The heteromorphic contact with a limited strip area caused by the local geometry of an object transmits primarily two-dimensional friction moments $\left\{\mathbf{q}_{\mathbf{n}}\right.$, $\left.\mathbf{q}_{\mathbf{t}}\right\}$ in addition to a three-dimensional force $\left\{\mathbf{p}_{\mathbf{n}}, \mathbf{p}_{\mathbf{t}}, \mathbf{p}_{\mathbf{o}}\right\}$. The transmissible force and friction moment are shown in Fig. 5. The area of oblique lines indicates the contact area on the soft fingertip in Fig. 5. $\mathbf{q}_{\mathbf{n}}$ is parallel to the surface normal $\mathbf{n}$ of the fingertip at the contact area. Here, $\mathbf{q}_{\mathbf{n}}$ is referred to as the normal component of the friction moment. $\mathbf{q}_{\mathbf{t}}$ is perpendicular to both the surface normal $\mathbf{n}$ and the orientation of the stick-like object. We call $\boldsymbol{q}_{\boldsymbol{t}}$ the tangential component of the friction moment. Then, we refer to this contact as soft line contact.


Fig. 5. Soft line contact.

When a typical hard spherical fingertip contacts an object, the contact always transmits only a three-dimensional force $\mathbf{p}_{\mathbf{n}}, \mathbf{p}_{\mathbf{t}}, \mathbf{p}_{\mathbf{o}}$. Features of the soft fingertip mentioned above provide more capability of tactile sensing than a hard fingertip.

## B. Measurement of Edge Direction

Measurement of the orientation of a stick-like object is formalized as follows. The geometrical relationship for the soft line contact is shown in Fig. 6. Let $B_{x y z}, \mathbf{f}$, and $\mathbf{m}$ be the reference frame, the measured force, and the measured moment, respectively. Let $\mathbf{r}, \mathbf{n}, \mathbf{p}$, and $\mathbf{q}$ be the contact location on a soft fingertip, the surface normal at $\mathbf{r}$, the contact force at $\mathbf{r}$, and the friction moment at $\mathbf{r}$, respectively. Each vector is expressed with respect to $B_{x y z}$. The reference frame $B_{x y z}$ is attached to the six-axis force/moment sensor reference frame in our implementation. The force $\mathbf{f}$ and moment $\mathbf{m}$ exerted by the fingertip are measured by the sixaxis force/moment sensor. The position $\mathbf{r}$ of the contact point on the fingertip is calculated using Bicchi's method [8]. The
friction moment $\mathbf{q}$ is obtained from the balance equation of force and moment as:

$$
\begin{gather*}
\mathbf{f}=\mathbf{p}  \tag{11}\\
\mathbf{m}=\mathbf{q}+\mathbf{r} \times \mathbf{p} \tag{12}
\end{gather*}
$$

When the contact is a soft line contact, $\mathbf{q}$ consists of $\mathbf{q}_{\mathbf{n}}$ and $\mathbf{q}_{\mathbf{t}}$, where $\mathbf{q}_{\mathbf{t}}$ is the tangential component of the friction moment, which is additionally transmitted to the fingertip due to the penetration of the stick like object into the soft fingertip. Then, $\mathbf{q}_{\mathbf{t}}$ is obtained from the friction moment $\mathbf{q}$ measured at the contact by subtracting $\mathbf{q}_{\mathbf{n}}$, as shown in Eq. (14).

We can measure the orientation of the stick-like object as the outer product of $\mathbf{n}$ and $\mathbf{q}_{\mathbf{t}} \cdot \mathbf{n}$ and $\mathbf{q}_{\mathbf{t}}$ are expressed as:

$$
\begin{align*}
\mathbf{n} & =\frac{\nabla S(\mathbf{r})}{\|\nabla S(\mathbf{r})\|}  \tag{13}\\
\mathbf{q}_{\mathbf{t}} & =\mathbf{q}-(\mathbf{q} \cdot \mathbf{n}) \mathbf{n}, \tag{14}
\end{align*}
$$

where $\nabla$ is the gradient operator, and $S(\mathbf{r})$ is the function describing the surface of the fingertip.


Fig. 6. Geometric relationship for soft line contact.

## IV. EXPERIMENTS

## A. Experimental Setup

The experimental setup is shown in Fig. 7. An object composed of two-links connected by one joint is on the table as shown in Fig. 8. The left grip of the object is fixed to the table. The right grip can be freely rotated. The rotation axis is estimated through the tactile sensing between the soft fingertip and the right grip. We have developed a robotic fingertip with a soft skin. Fig. 9 shows the inner structure and its appearance. The fingertip is composed of an inner shell and a soft skin. The inner shell is made of aluminum. The inner shell is a cylinder with a hemisphere on its top. Its radius is 11 mm . The soft skin is made of silicone rubber covering the inner shell. The skin has constant thickness. The thickness of the skin is 5 mm . The inner shell is fixed to a six-axis force/torque sensor (BL AUTOTEC, LTD. NANO 5/4). The exerted force/moment to the fingertip is measured by the force/torque sensor. The soft fingertip is connected to the probe of the 3-D coordinate measuring machine (microcord C604, Mitutoyo Corp.). The relative position and orientation between the 3-D coordinate measuring machine coordinate system and the fingertip coordinate system are
known. Then, both the location of a contact point and the direction of an edge measured in the fingertip coordinate system can be expressed in the 3-D coordinate measuring machine coordinate system.


Fig. 7. Experimental setup.


Fig. 8. Object composed of two-links connected by one joint.


Fig. 9. Inner structure of a robotic fingertip with a soft skin and its appearance.

## B. Formalization for experiments

1) Proposed method: In the experiments, the robotic fingertip rotates the right grip on $x-y$ plane (Fig. 10). Therefore, let $\boldsymbol{x}_{\boldsymbol{i}}=\left(\begin{array}{lll}x_{i} & y_{i}\end{array}\right)^{T}, \boldsymbol{p}=\left(\begin{array}{ll}p & q\end{array}\right)^{T}, \boldsymbol{e}_{\boldsymbol{i}}=\left(\begin{array}{ll}e_{x i} & e_{y i}\end{array}\right)^{T}$, and $\boldsymbol{u}=\left(\begin{array}{lll}0 & 0 & 1\end{array}\right)^{T}$ be substituted in the formalization of section II. Then, $s_{d}=0$ and $t_{d}=-\left(\boldsymbol{x}_{\boldsymbol{i}}-\boldsymbol{p}\right) \cdot \boldsymbol{e}_{\boldsymbol{i}}=$ $-e_{x i}\left(x_{i}-p\right)-e_{y i}\left(y_{i}-q\right)$ are obtained. By using these $s_{d}$ and $t_{d}$,

$$
\left|\begin{array}{l}
\left(1-e_{x i}^{2}\right)\left(x_{i}-p\right)-e_{x i} e_{y i}\left(y_{i}-q\right)  \tag{15}\\
\left(1-e_{y i}^{2}\right)\left(y_{i}-q\right)-e_{x i} e_{y i}\left(x_{i}-p\right)
\end{array}\right|=d
$$

is obtained. Since $\boldsymbol{e}_{\boldsymbol{i}}$ is a unit vector,

$$
\begin{equation*}
e_{y i}\left(x_{i}-p\right)-e_{x i}\left(y_{i}-q\right)=d \tag{16}
\end{equation*}
$$

is obtained from equation (15). By rearranging equation (16) about $p, q$, and $d$,

$$
\left(\begin{array}{ll}
e_{y i} & -e_{x i} 1
\end{array}\right)\left(\begin{array}{l}
p  \tag{17}\\
q \\
d
\end{array}\right)=e_{y i} x_{i}-e_{x i} y_{i}
$$

is obtained. $N$ equations are obtained by measuring the location of a contact point and the direction of an edge of the object $N$ times. We express the $N$ equations by rearranging them as:

$$
\left(\begin{array}{ccc}
e_{y 1} & -e_{x 1} & 1  \tag{18}\\
\vdots & \ddots & \vdots \\
e_{y N} & -e_{x N} & 1
\end{array}\right)\left(\begin{array}{c}
p \\
q \\
d
\end{array}\right)=\left(\begin{array}{c}
e_{y 1} x_{1}-e_{x 1} y_{1} \\
\vdots \\
e_{y N} x_{N}-e_{x N} y_{N}
\end{array}\right)
$$

We express this equation as:

$$
\begin{equation*}
A x=b \tag{19}
\end{equation*}
$$

where $\boldsymbol{A}$ is a $N$-by- 3 matrix. Equation (19) can be solved using the pseudo-inverse matrix $\boldsymbol{A}^{+}$of the matrix $\boldsymbol{A}$ as follows:

$$
\begin{align*}
\boldsymbol{x} & =\boldsymbol{A}^{+} \boldsymbol{b}+\left(\boldsymbol{I}-\boldsymbol{A}^{T} \boldsymbol{A}\right) \boldsymbol{\xi}  \tag{20}\\
\boldsymbol{A}^{+} & =\left(\boldsymbol{A}^{T} \boldsymbol{A}\right)^{-1} \boldsymbol{A}^{T} \tag{21}
\end{align*}
$$

where $\boldsymbol{I}$ is the identity matrix, $\boldsymbol{\xi}$ is an arbitrary vector. $\boldsymbol{x}$ which brings minimum $\|\boldsymbol{A} \boldsymbol{x}-\boldsymbol{b}\|$ can be calculated by using $\boldsymbol{A}^{+}$. The position of the joint-axis of the object on $x-y$ plane $(p, q)$ is estimated as this $\boldsymbol{x}$.


Fig. 10. Proposed method: Estimation method of the position of the jointaxis $(p, q)$ based on measurement of the locations of contact points ( $\boldsymbol{x}_{\boldsymbol{i}}$, $\boldsymbol{x}_{\boldsymbol{j}}, \boldsymbol{x}_{\boldsymbol{k}}$ ) and the directions of edges of an object ( $\boldsymbol{e}_{\boldsymbol{i}}, \boldsymbol{e}_{\boldsymbol{j}}, \boldsymbol{e}_{\boldsymbol{k}}$ ).
2) Simple method: For comparison, we show another estimation method of the position of a joint-axis of an object. In this method, we assume that a contact point does not move on the surface of the object during a rotation of the joint of the object. We call this method simple method.

The distance between a contact point on the object and the joint-axis of the object is constant during the rotation of the revolute joint. Therefore, the position of the joint-axis is on the perpendicular bisector of a line segment that is bounded by two contact points before and after the rotation of the joint. So, the position of the joint-axis can be estimated
as the location of the intersection point of more than one perpendicular bisector (Fig. 11). The location of a contact point is measured $N$ times during the rotation of the joint. The perpendicular bisector is expressed as:

$$
\begin{equation*}
y=-\frac{x_{j}-x_{i}}{y_{j}-y_{i}}\left(x-\frac{x_{i}+x_{j}}{2}\right)+\frac{y_{i}+y_{j}}{2},\left(y_{i} \neq y_{j}\right) \tag{22}
\end{equation*}
$$

where $\left(x_{i} y_{i}\right)$ and $\left(x_{j} y_{j}\right)$ are the locations of the contact points at the $i$ th and $j$ th measurements, respectively. By rearranging this equation about $x$ and $y$,

$$
\begin{equation*}
\left(x_{j}-x_{i} y_{j}-y_{i}\right)\binom{x}{y}=\frac{1}{2}\left(x_{j}^{2}-x_{i}^{2}+y_{j}^{2}-y_{i}^{2}\right) \tag{23}
\end{equation*}
$$

is obtained. ${ }_{N} \mathrm{C}_{2}$ similar equations are obtained by measuring the location of a contact point $N$ times. We express the ${ }_{N} \mathrm{C}_{2}$ equations by rearranging them as:

$$
\begin{gather*}
\left(\begin{array}{cc}
x_{2}-x_{1} & y_{2}-y_{1} \\
\vdots & \vdots \\
x_{N}-x_{N-1} & y_{N}-y_{N-1}
\end{array}\right)\binom{x}{y} \\
=\left(\begin{array}{c}
\frac{1}{2}\left(x_{2}^{2}-x_{1}^{2}+y_{2}^{2}-y_{1}^{2}\right) \\
\vdots \\
\frac{1}{2}\left(x_{N}^{2}-x_{N-1}^{2}+y_{N}^{2}-y_{N-1}^{2}\right)
\end{array}\right) \tag{24}
\end{gather*}
$$

We express this equation as:

$$
\begin{equation*}
A x=b \tag{25}
\end{equation*}
$$

where $\boldsymbol{A}$ is a ${ }_{N} \mathrm{C}_{2}$-by- 3 matrix. Same as section IV-B.1), the position of the joint-axis on $x-y$ plane $(p, q)$ is estimated by using the pseudo-inverse matrix $\boldsymbol{A}^{+}$of the matrix $\boldsymbol{A}$ as follows:

$$
\begin{equation*}
x=A^{+} b \tag{26}
\end{equation*}
$$

where $\boldsymbol{x}=(x y)^{T}=(p q)^{T}$.


Fig. 11. Simple method: Estimation method of the position of the jointaxis $(p, q)$ based on measurement of the locations of contact points ( $\boldsymbol{x}_{\boldsymbol{i}}$, $\boldsymbol{x}_{\boldsymbol{j}}, \boldsymbol{x}_{\boldsymbol{k}}$ ).

## C. Sequence of experiment

The sequence of the experiment is as follows:

1) The robotic fingertip touches the right grip of the object.
2) The location of the contact point and the direction of the edge of the object are measured.
3) The robotic fingertip is moved away from the right grip. Then, the right grip is rotated counterclockwise by human hand by 30 degrees.
4) The location of a contact point and the direction of an edge are measured three times by repeating from 1) to 3). Then, the position of the joint-axis of the object is estimated based on the measurement of the contact information.
At the step 2), the contact information is measured when the force of $5[\mathrm{~N}]$ is exerted for penetration of the edge into the soft fingertip.

## D. Experimental Result

We investigated the influence of the movement of the contact point on the edge. We estimated the position of the joint-axis of the object in two cases with different conditions. First case is that the contact point moves on the edge of the right grip. The other case is that the contact point does not move on the edge. We estimated the position of the joint-axis ten times in each case. The estimation results are shown in TABLE I. Results in the case that the contact points move are shown in Fig. 12. Results in the case that the contact points do not move are shown in Fig. 13. The left images are results of the proposed method. The right images are results of the simple method. The blue circle indicates the true value of the position of the joint-axis $(p, q)$. The true value of $(p, q)$ is $(17 \mathrm{~mm}, 105 \mathrm{~mm})$. The red x indicates the estimated position of the joint-axis. The green squares indicate the measured locations of the contact points. The green lines indicate the measured directions of the edges of the right grip.

TABLE I
THE ESTIMATION RESULTS OF THE POSITION OF THE JOINT-AXIS $(p, q)$.

| The estimation results of the position of the joint-axis $(p, q)[\mathrm{mm}]$ |  |  |  |
| :---: | :---: | :---: | :---: |
| in the case that <br> the contact points move | in the case that <br> the contact points do not move |  |  |
| proposed method | simple method | proposed method | simple method |
| $(21,99)$ | $(18,76)$ | $(23,109)$ | $(13,100)$ |
| $(24,109)$ | $(21,79)$ | $(19,103)$ | $(14,101)$ |
| $(20,105)$ | $(22,79)$ | $(23,109)$ | $(6,100)$ |
| $(25,108)$ | $(22,78)$ | $(25,114)$ | $(20,101)$ |
| $(21,108)$ | $(21,77)$ | $(22,111)$ | $(19,102)$ |
| $(17,100)$ | $(20,76)$ | $(21,105)$ | $(21,101)$ |
| $(22,111)$ | $(18,76)$ | $(27,115)$ | $(21,101)$ |
| $(20,103)$ | $(21,79)$ | $(19,105)$ | $(19,101)$ |
| $(20,104)$ | $(19,76)$ | $(26,113)$ | $(14,100)$ |
| $(24,110)$ | $(25,80)$ | $(19,105)$ | $(13,101)$ |
| The average error of the estimation results of the position of the joint-axis [mm] |  |  |  |
|  |  |  |  |

The position of the joint-axis was successfully estimated by using the proposed method in both cases. The error of the estimation depends on the accuracy of the tactile measurement including the location of the contact point and the direction of the edge. Repetition of tactile measurements will improve the accuracy of the estimation. In contrast, the estimation error of the simple method is large in the case that the contact points move (in Fig. 12). The movement of the contact point causes the increase of the error in the case of the simple method.


Fig. 12. Estimation results in the case that the contact points move.


Fig. 13. Estimation results in the case that the contact points do not move.

## V. Conclusion

We propose a tactile sensing of estimating the position and orientation of a joint-axis of an object. This tactile sensing is based on the measurement of both the position of a contact point and the direction of an edge of an object by using a robotic fingertip with a six-axis force/torque sensor and a soft skin. The estimation of a joint-axis of a linked object is demonstrated by using the soft fingertip. The tactile sensing is robust against not only failure factors with the vision sensing but also the movement of a contact point on the surface of the object.

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