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A Riemannian-Geometry Approach for Dynamics and Control of Object Manipulation under Constraints

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***Abstract*— A Riemannian-geometry approach for control and stabilization of dynamics of object manipulation under holonomic or non-holonomic (but Pfaffian) constraints is presented. First, position/force hybrid control of an endeffector of a multi-joint redundant (or nonredundant) robot under a nonholonomic constraint is reinterpreted in terms of "submersion" in Riemannian geometry. A force control signal constructed in the image space spanned from the constraint gradient can be regarded as a lifting in the direction orthogonal to the kernel space. By means of the Riemannian distance on the constraint submanifold, stability on a manifold for a redundant system under holonomic constraints is discussed. Second, control and stabilization of dynamics of two-dimensional object grasping and manipulation by using a pair of multi-joint robot fingers are tackled, when a rigid object is given with arbitrary shape. Then, it is shown that rolling contact constraint induce the Euler equation of motion in an implicit function form, in which constraint forces appear as wrench vectors affecting on the object. The Riemannian metric can be introduced in a natural way on a constraint submanifold induced by rolling contacts. A control signal called "blind grasping" is defined and shown to be effective in stabilization of grasping without using the details of information of object shape and parameters or external sensing. The concept of stability of the closed-loop system under constraints is renewed in order to overcome the degrees-of-freedom redundancy problem. An extension of Dirichlet-Lagrange's stability theorem to a system of DOF-redundancy under constraints is presented by using a Morse-Lyapunov function.**

I. INTRODUCTION

Among roboticsists it is implicitly known that robot motions can be interpreted in terms of orbits on a high-dimensional torus or trajectories in an n-dimensional configuration space. Planning of robot motions has been developed traditionally on the basis of kinematics on a configuration space as an n-dim. numerical space $\boldsymbol{R}^n$ [1].

This paper first emphasizes a mathematical observation that, given a robot as a multi-body mechanism with n degrees-of-freedom whose endpoint is free, the set of its all postures can be regarded as a Riemannian manifold (M, g) associated with the Riemannian metric g that constitutes the robot's inertia matrix. A geodesic connecting any two postures can correspond to an orbit expressed on a local coordinate chart and generated by a solution to the Euler-Lagrange equation of robot motion that originates only from inertial force [2]. This Riemannian-geometry viewpoint is extended in this paper to an important class of multi-body dynamics physically interacting with an object or with environment through holonomic or/and nonholonomic (but Pfaffian) constraints. Holonomic constraints are defined as a set of infinitely differentiable functions from a product manifold of multi-body Riemannian manifolds onto an open set of a 2 or 3 dimensional Euclidean space called the task space. Such a mapping can be treated as a submersion from the product Riemannian manifold to m(= 2 or 3)-dimensional Euclidean space. Hence, holonomic constraints induce a Riemannian submanifold with a naturally induced metric. An Euler-Lagrange equation is formulated under such constraints. Rolling contact constraints between rigid bodies are also introduced in a similar but more careful manner by defining a Pfaffian connection in the sense of Cartan among two adjacent smooth curves (when $m = 2$) or surfaces (when $m = 3$). In the case of $m = 2$, an Euler-Lagrange equation together with a kinematic equation can be formulated in an implicit or explicit form.

Based upon these observations, the well-known methodology of hybrid (position/force) control for a robot whose end effector is constrained on a surface is re-examined and shown to be effective even if the robot is of redundancy in degrees-of-freedom. An extension of modeling dynamics of 2-dimensional object manipulation by means of a pair of robot fingers with rolling contacts when the shape of an object is arbitrarily given. A coordinated control signal called "blind grasping" is proposed and shown to be effective in realizing stable grasping.

II. RIEMANNIAN MANIFOLD: A SET OF ALL POSTURES

Lagrange's equation of motion of a multi-joint system with 2 DOFs (Degrees-Of-Freedom) shown in Fig. 1 is described by the formula

$$H(q)\ddot{q} + \left\{ \frac{1}{2}\dot{H}(q) + S(q,\dot{q}) \right\} \dot{q} + g(q) = u \tag{1}$$

where $q = (q_1, q_2)^{\mathrm{T}}$ denotes the vector of joint angles, $H(q)$ the inertia matrix, and $S(q,\dot{q})\dot{q}$ the gyroscopic force term including centrifugal and Coriolis forces, $g(q)$ the gradient vector of a potential function $P(q)$ due to gravity with respect to q, and u the joint torque generated by joint actuators [3]. It is well known that the inertia matrix $H(q)$ is symmetric and positive definite and there exist a positive

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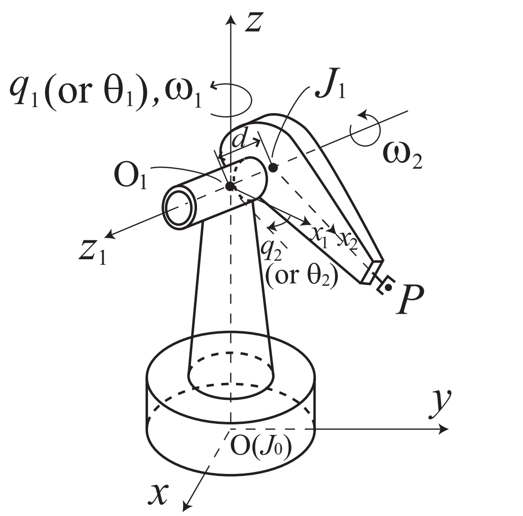

Fig. 1. A 2-DOF part of vertically-revolute robot PUMA-560

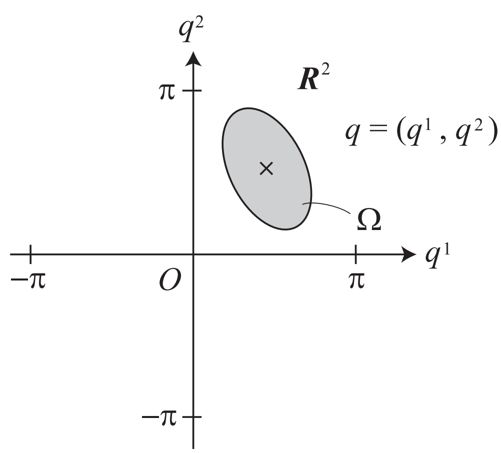

Fig. 2. A homeomorphism $\phi : U \to \Omega$ is called a chart

constant h_m together with a positive definite constant diagonal matrix H_0 such that

$$h_m H_0 \leq H(q) \leq H_0 \tag{2}$$

for any q. It should be also noted that $S(q, \dot{q})$ is skew symmetric and linear and homogeneous in $\dot{q}$. More in detail, the ij-th entry of $S(q, \dot{q})$ denoted by s_{ij} can be described in the form [3]:

$$s_{ij} = \frac{1}{2}\left\{\frac{\partial}{\partial q_j}\left(\sum_{k=1}^{n}\dot{q}_k h_{ik}\right) - \frac{\partial}{\partial q_i}\left(\sum_{k=1}^{n}\dot{q}_k h_{jk}\right)\right\} \tag{3}$$

where $H(q) = (h_{ij}(q))$, from which it follows apparently that $s_{ij} = -s_{ji}$. Since we assume that the objective system to be controlled is a series of rigid links serially connected through rotational joints with single DOF, each entry of $H(q)$ is a constant or a sinusoidal function of components of joint angle vector q. That is, each element of $H(q)$ and $g(q)$ is differentiable of class C^∞ (infinitely differentiable in q).

When two joint angles θ_1 and θ_2 are given in $\theta_i \in (-\pi, \pi)$, $i = 1, 2$, for the 2 DOF robot arm shown in Fig. 1, the posture

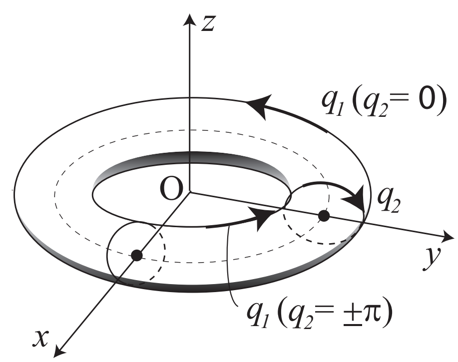

Fig. 3. A two dimensional torus is expressed by $T^2 = S^1 \times S^1$

$p(\theta_1, \theta_2)$ is determined naturally. Denote the set of all such possible postures by M and introduce a family of subsets of M such that, for any $p \in M$ with joint angles $p = (\theta_1, \theta_2)$ and any number $\alpha > 0$, a set of all $p' = (\theta_1', \theta_2')$ defined as

$$U_{p,\alpha} = \{p' : \|p' - p\|_H < \alpha\} \tag{4}$$

where

$$\|p' - p\|_H = \sqrt{\sum_{i,j} h_{ij}(p)(\theta_i' - \theta_i)(\theta_j' - \theta_j)} \tag{5}$$

can be regarded as an open subset of M. Then, the set M with such a family of open subsets can be regarded as a topological manifold. It is possible to show that the manifold M becomes Hausdorff and compact. Further, every point p of M has a neighborhood U that is homeomorphic to an open subset Ω of 2-dimensional numerical space $\boldsymbol{R}^2$. Such a homeomorphism $\phi : U \to \Omega$ is called a coordinate chart. In fact, a neighborhood $U_{p,\alpha}$ of posture p with joint angles (θ_1, θ_2) in Fig. 1 can be mapped to an open set Ω in $\boldsymbol{R}^2$ with 2-dimensional numerical coordinates (q^1, q^2) with the origin O (Fig. 2). In this case, it is possible to see that the original set M of robot postures can be visualised as a torus shown in $\boldsymbol{R}^3$ (see Fig. 3) in which angles q_1 and q_2 are defined. It is quite fortunate to see that, in the case of typical robots like the one shown in Fig. 1, the local coordinates $(q^1, \cdots, q^n)$ can be identically chosen as a set of n independent joint angles $(\theta_1, \cdots, \theta_n)$ by setting $q^i = \theta_i$ $(i = 1, \cdots, n)$. It is also interesting to see that the torus in Fig. 3 is made to be topologically coincident with the set of all arm endpoints $P = (x, y, z)$. As discussed in detail in mathematical text books [4], [5], the topological manifold (M, p) of such a torus can be regarded as a differentiable manifold of class C^∞.

Now, it is necessary to define a tangent vector to an abstract differentiable manifold M at $p \in M$. Let I be an interval $(-\epsilon, \epsilon)$ and define a curve $c(t)$ by a mapping $c : I \to M$ such that $c(0) = p$. A tangent vector to M at p is an equivalence class of curves $c : I \to M$ for the equivalence relation $\sim$ defined by

$c \sim \bar{c}$ if and only if, in a coordinate chart (U, ϕ) around p, we have $(\phi \circ \bar{c})'(0) = (\phi \circ c)'(0)$, where symbol $(\ )'$ means differentiation of $(\ )$ with respect to $t \in I$. This definition of tangent vectors to M at p does not depend on choice of the coordinate chart at p, as discussed in text books [4], [5]. Let us denote the set of all tangent vectors to M at p by T_pM and call it the tangent space at $p \in M$. It has an n-dimensional linear space structure like $\boldsymbol{R}^n$. We also denote the disjoint union of the tangent spaces to M at all the points of M by TM and call it the tangent bundle of M.

Now, we are in a position to define a Riemannian metric on a differentiable manifold (M, p) as a mapping $g_p : T_pM \times T_pM \to \boldsymbol{R}$ such that $p \to g_p$ is of class C^∞ and $g_p(u, v)$ for $u \in T_pM$ and $v \in T_pM$ is a symmetric positive definite quadratic form:

$$g_p(u, v) = \sum_{i,j=1}^{n} g_{ij}(p) u^i u^j \tag{6}$$

Suppose that M is a connected Riemannian manifold. If $c : I[a, b] \to M$ is a curve segment of class C^∞, we define the length of c to be

$$L(c) = \int_a^b \|\dot{c}(t)\| \mathrm{d}t = \int_a^b \sqrt{g_{c(t)}(\dot{c}(t), \dot{c}(t))}\ \mathrm{d}t \tag{7}$$

where we assume $\dot{c}(t) \neq 0$ for any $t \in I$ and call such a curve segment to be regular. A mapping of class C^∞ $c : [a, b] \to M$ is called a piecewise regular curve segment (for brevity, we call it an admissible curve) if there exists a finite subdivision $a = a_0 < a_1 < \cdots < a_k = b$ such that $c(t)$ for $t \in [a_{i-1}, a_i]$ is a regular curve for $i = 1, \cdots, k$. Then, it is possible to define for any pair of points $p, p' \in M$ the Riemannian distance $d(p, p')$ to be the infimum of all admissible curves from p to q. It is well known [4], [5] that, with the distance function d defined above, any connected Riemannian manifold becomes a metric space whose induced topology is coincident with the given manifold topology. An admissible curve c in a Riemannian manifold is said to be minimizing if $L(c) \leq L(\tilde{c})$ for any other admissible curve $\tilde{c}$ with the same endpoints. It follows immediately from the definition of distance that c is minimizing if and only if $L(c)$ is equal to the distance between its endpoints. Further, it is known that if the Riemannian manifold $\{M, g\}$ is complete then for any pair of points p and p' there exists at least a minimizing curve $c(t)$, $t \in [a, b]$, with $c(a) = p$ and $c(b) = p'$. If such a minimizing curve $c(t)$ is described with the aid of coordinate chart (U, ϕ) as $\phi(c(t)) = (q^1(t), \cdots, q^n(t))$ then $q(t) = \phi(c(t))$ satisfies the 2nd-order differential equation:

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} q^k(t) + \sum_{i,j=1}^{n} \Gamma_{ij}^k(c(t)) \frac{\mathrm{d}q^i(t)}{\mathrm{d}t} \frac{\mathrm{d}q^j(t)}{\mathrm{d}t} = 0 \qquad (k = 1, \cdots, n) \tag{8}$$

where Γ_{ij}^k denotes Christoffel's symbol defined by

$$\Gamma_{ij}^k = \frac{1}{2} \sum_{h=1}^{m} g^{kh} \left(\frac{\partial g_{ih}}{\partial q^j} + \frac{\partial g_{jh}}{\partial q^i} - \frac{\partial g_{ij}}{\partial q^h} \right) \tag{9}$$

and (g^{kh}) denotes the inverse of matrix (g_{kh}). A curve $q(t) : I \to U$ satisfying (8) together with $\phi^{-1}(q(t))$ is called a geodesic and (8) itself is called the Euler-Lagrange equation or the geodesic equation.

Given a C^∞-class curve $c(t) = I[a, b] \to M$, the quantity

$$E(c) = \frac{1}{2} \int_a^b \|\dot{c}(t)\|^2\ \mathrm{d}t = \frac{1}{2} \int_a^b g_{c(t)}(\dot{c}(t), \dot{c}(t))\ \mathrm{d}t \tag{10}$$

is called the energy of the curve. Then, by applying the Cauchy-Schwartz inequality for (7) we have

$$L(c)^2 \leq 2(b - a) E(c) \tag{11}$$

Further, the equality of (11) follows if and only if $\|\dot{c}(t)\|$ is constant. It is also possible to see that if $c(t)$ is a geodesic with $c(a) = p$ and $c(b) = p'$ then for any other curve $\gamma(t)$ with the same endpoints it holds

$$E(c) = \frac{L(c)^2}{2(b - a)} \leq \frac{L(\gamma)^2}{2(b - a)} \leq E(\gamma) \tag{12}$$

The equalities hold if and only if $\gamma(t)$ is also a geodesic. Conversely, if $c(t)$ with $c(a) = p$ and $c(b) = p'$ is a C^∞ curve that minimizes the energy and makes $g_{c(t)}(\dot{c}(t), \dot{c}(t))$ constant, then $c(t)$ becomes a geodesic connecting $c(a) = p$ and $c(b) = p'$.

III. Riemannian Geometry of Robot Dynamics

Dynamics of a robotic mechanism with n rigid bodies connected in series through rotational joints are described by Lagrange's equation of motion, as shown in (1). It is implicitly assumed that the axis of rotation of the first body is fixed in an inertial frame and denoted by z-axis that is perpendicular to the xy-plane as shown in Fig. 1. If there is no gravity force affecting motion of the robot and no external force, that is, each joint is free to rotate, then the equation of motion of the robot can be described as

$$H(q)\ddot{q} + \left\{ \frac{1}{2} \dot{H}(q) + S(q, \dot{q}) \right\} \dot{q} = u \tag{13}$$

This case is valid for motions of a revolution-joint robot as in Fig. 1 if it is located in weightless environment on an artificial satellite. In general, we can represent a posture p of the robot as a physical entity by a family of joint angles θ_i $(i = 1, \cdots, n)$, which can be expressed by a point $\Theta = (\theta_1, \cdots, \theta_n)$ in the n-dimensional numerical space $\boldsymbol{R}^n$. In other words, we can naturally have an isomorphism $\phi : U \to \Omega$, where $U \subset M$ and Ω is an open subset of $\boldsymbol{R}^n$. In other words, a local coordinate chart $\phi(U)$ $(= \Omega)$ can be treated to be identical to U itself, an open subset of M, by regarding $q = (q^1, \cdots, q^n)^{\mathrm{T}}$ ("T" denotes transpose and hence q a column vector) identical to Θ by setting $q^i = \theta_i$ $(i = 1, \cdots, n)$. That is, in this way the abstract manifold M as the set of all robot postures should be regarded as an n-dimensional torus T^n as an n-tuple direct product of $S^1 : T^n = S^1 \times \cdots \times S^1$. Hence, a robot posture $p \in M$ can be represented by a point Θ in T^n and also expressed by a joint vector q in $\boldsymbol{R}^n$.

From the definition of inertia matrices, $H(q)$ is symmetric and positive definite and the kinetic energy is expressed as a quadratic form:

$$K(q,\dot{q}) = \frac{1}{2}\dot{q}^{\mathrm{T}}H(q)\dot{q} \tag{14}$$

Hence, the equation of motion of the robot arm is expressed by Lagrange's equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{\partial}{\partial \dot{q}}L(q,\dot{q})\right\} - \frac{\partial}{\partial q}L(q,\dot{q}) = u \tag{15}$$

where $L(q,\dot{q}) = K(q,\dot{q})$ and u stands for a generalized external force vector. It is interesting to note that in differential geometry (15) can be described as

$$\sum_i h_{ki}\ddot{q}^i + \sum_{i,j}\Gamma_{ikj}(q)\dot{q}^i\dot{q}^j = u^k \tag{16}$$

where Γ_{ikj} denotes Christoffel's symbol of the first kind defined by

$$\Gamma_{ikj} = \frac{1}{2}\left(\frac{\partial h_{jk}}{\partial q^i} + \frac{\partial h_{ik}}{\partial q^j} - \frac{\partial h_{ij}}{\partial q^k}\right) \tag{17}$$

For later use, we introduce another Christoffel's symbol called the second kind as shown in the following

$$\begin{aligned}\Gamma_{ij}^k &= \frac{1}{2}\sum_{l=1}^n h^{lk}\left(\frac{\partial h_{jl}}{\partial q^i} + \frac{\partial h_{il}}{\partial q^j} - \frac{\partial h_{ij}}{\partial q^l}\right) \\ &= \frac{1}{2}\sum_{l=1}^n h^{lk}\Gamma_{ilj}\end{aligned} \tag{18}$$

where (h^{lk}) denotes the inverse of (h_{lk}), the inertia matrix $H(q) = (h_{lk})$. Since (h_{kl}) and (h^{kl}) are symmetric, it follows that $\Gamma_{ikj} = \Gamma_{jki}$ and $\Gamma_{ij}^k = \Gamma_{ji}^k$. Now, we show that eq.(13) is equivalent to (16) by bearing in mind that $\dot{H}(q) = \sum_i\{\partial H(q)/\partial q^i\}\dot{q}^i$ and the skew-symmetric matrix $S(q,\dot{q})$ is expressed as in (3). In fact, the second term in the bracket ( ) of (17) corresponds to the first term in { } of (3) and the third term of (17) does to the second term in { } of (3). Hence, it follows from (3) that

$$\begin{aligned}\sum_{j=1}^n s_{kj}\dot{q}^j &= \sum_{j=1}^n \frac{1}{2}\left[\left\{\frac{\partial}{\partial q^j}\left(\sum_{i=1}^n \dot{q}^i h_{ki}\right)\right\}\dot{q}^i \right. \\ &\qquad \left. - \left\{\frac{\partial}{\partial q^k}\left(\sum_{i=1}^n \dot{q}^i h_{ij}\right)\right\}\dot{q}^j\right] \\ &= \sum_{j=1}^n\sum_{i=1}^n \frac{1}{2}\left\{\left(\frac{\partial h_{ik}}{\partial q^j} - \frac{\partial h_{ij}}{\partial q^k}\right)\right\}\dot{q}^i\dot{q}^j\end{aligned} \tag{19}$$

Substituting this into (16) by comparing the last two terms of (17) with the last bracket { } of (19) results in the equivalence of (13) to (16). It is easy to see that multiplication of (16) by $H^{-1}(q)$ yields

$$\ddot{q}^k + \sum_{i,j}\Gamma_{ij}^k\dot{q}^i\dot{q}^j = \sum_j h^{kj}u^j, \quad k = 1,\cdots,n \tag{20}$$

If $u = 0$, this expression is nothing but the Euler-Lagrange equation shown in (8). By this reason, from now on we use symbol $g_{ij}(q)$ instead of $h_{ij}(q)$ for the inertia matrix $H(q)$ even when robot dynamics are treated.

Now, on the abstract topological manifold M as a set of all possible postures of a robot, suppose that a Riemannian metric is given by a scalar product on each tangent space T_pM:

$$< v, w >= g_{ij}(p)v^iw^j \tag{21}$$

where $v = v^i(\partial/\partial q^i) \in T_pM$ and $w = w^j(\partial/\partial q^j) \in T_pM$, and the summation symbol $\sum$ in i and j is omitted, and $q = (q^1,\cdots,q^n)$ represents local coordinates. Then, the manifold (M,p) can be regarded as a Riemannian manifold and becomes complete as a metric space. Then, according to the Hopf-Rinow theorem [4], any two points $p, q \in M$ can be joined by a geodesic of length $d(p,q)$, that is, a curve satisfying (8) with shortest length, where

$$\begin{aligned}d(p,q) &= \inf\int_a^b \|\dot{c}(t)\|\ \mathrm{d}t \\ &= \inf\int_a^b \sqrt{<\dot{c}(t),\dot{c}(t)>}\ \mathrm{d}t\end{aligned} \tag{22}$$

with $c(a) = p$ and $c(b) = q$.

As discussed in the previous section, geodesics are the critical points of the energy functional $E(c)$. Further, a geodesic curve $c(t)$ satisfies $\|\dot{c}(t)\| = \mathrm{const.}$ In fact, by regarding $c(t) = q(t)$ that is an orbit on Ω, we have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t} < \dot{q},\dot{q} > &= \frac{\mathrm{d}}{\mathrm{d}t}\left(g_{ij}(q(t))\dot{q}_i(t)\dot{q}_j(t)\right) \\ &= \frac{\mathrm{d}}{\mathrm{d}t}\left\{\dot{q}^{\mathrm{T}}(t)G(q(t))\dot{q}(t)\right\} \\ &= \ddot{q}^{\mathrm{T}}(t)G(q)\dot{q}(t) + \dot{q}^{\mathrm{T}}(t)\dot{G}(q)\dot{q}(t) + \dot{q}^{\mathrm{T}}(t)G(q)\ddot{q}(t) \\ &= 0\end{aligned} \tag{23}$$

where the equivalent expression to (13) with $u = 0$ is used. It is also important to note that, on a local coordinate chart $\Omega \subset \boldsymbol{R}^n$ corresponding to a neighborhood U of $p \in M$, an orbit $q(t)$ parameterized by time $t \in [a,b]$ and expressed by a solution to (20) (where u^j is of C^∞ in t) should satisfy

$$\sum_{j=1}^n \dot{q}^j(t)u^j(t) = \frac{\mathrm{d}}{\mathrm{d}t}E(q(t)) \tag{24}$$

or

$$\int_a^b\sum_{j=1}^n \dot{q}^j(t)u^j(t)\ \mathrm{d}t = E(q(b)) - E(q(a)) \tag{25}$$

as long as $q(t) \in \Omega$, where $E(q(t)) = (1/2) < \dot{q}(t),\dot{q}(t) >$. When $u(t) = 0$, $E(q(t)) = \mathrm{const.}$ and then the curve connecting $p = \phi^{-1}(q(a))$ and $p' = \phi^{-1}(q(b))$ must be a geodesic. In other words, an inertia-originated movement without being affected by the gravitational field or any external force field produces a geodesic orbit [2]. The most importantly, geodesics together with their lengths are invariant under any choice of local coordinates.

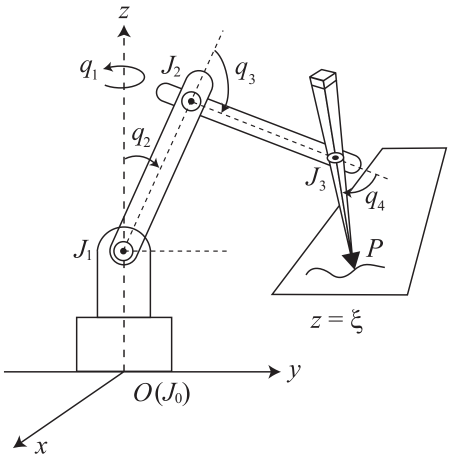

Fig. 4. A hand-writing robot with four DOFs whose endpoint P $(= (x, y))$ is constrained on a plane $z = \xi$

IV. Constraint Submanifold and Hybrid Position/Force Control

Consider a n-DOF robotic arm whose last link is a pencil and suppose that the endpoint of the pencil is in contact with a flat surface $\varphi(\boldsymbol{x}) = \xi$, where $\boldsymbol{x} = (x, y, z)^{\mathrm{T}}$. It is well-known that the Lagrange equation of motion of the systems is written as

$$G(q)\ddot{q} + \left\{\frac{1}{2}\dot{G} + S\right\}\dot{q} + g(q) = -\lambda\frac{\partial\varphi(\boldsymbol{x}(q))}{\partial q} + u \qquad (26)$$

where $\partial\varphi/\partial q$ can be decomposed into $\partial\varphi(q)/\partial q = J^{\mathrm{T}}(q)\partial\varphi/\partial\boldsymbol{x}$ and $J(q) = \partial\boldsymbol{x}/\partial q^{\mathrm{T}}$. On the constraint manifold $F_\xi = \{p | p \in M \text{ and } \varphi(\boldsymbol{x}(p)) = \xi\}$, consider a smooth curve $c(t) : I[a, b] \to F_\xi$ that connects the given two points $c(a) = p$ and $c(b) = p'$ where p and p' belong to F_ξ. The length of such a curve constrained to F_ξ is defined as

$$L(c) = \int_a^b \sqrt{g_{ij}(c(t))\dot{c}^i(t)\dot{c}^j(t)}\ \mathrm{d}t \qquad (27)$$

and consider the minimization

$$d(p, p') = \inf_{c \in F_\xi} L(c) \qquad (28)$$

that should be called the distance between p and p' on the constraint manifold. Then, the minimizing curve called the geodesic denoted identically by $q(t)$ $(= c(t))$ must satisfy the Euler equation

$$\ddot{q}^k(t) + \Gamma^k_{ij}\dot{q}^i(t)\dot{q}^j(t) = -\lambda(t)\cdot(\text{grad } \varphi(\boldsymbol{x}(t)))^k \qquad (29)$$

together with the constraint condition $\varphi(\boldsymbol{x}(t)) = \xi$, where

$$\text{grad } \varphi(\boldsymbol{x}(t)) = G^{-1}(q(t))J^{\mathrm{T}}(q)\frac{\partial\varphi}{\partial\boldsymbol{x}} \qquad (30)$$

and $J(q) = \partial\boldsymbol{x}^{\mathrm{T}}/\partial q$. It should be noted that from (29) and the inner product of (29) and $w = J^{\mathrm{T}}\partial\varphi/\partial\boldsymbol{x}$ it follows that

$$\sum_k\left(w^k\ddot{q}^k + w^k\Gamma^k_{ij}\dot{q}^i\dot{q}^j\right) = -\lambda\sum_k w^k g^{kj} w^j \qquad (31)$$

Since the holonomic constraint $\varphi(\boldsymbol{x}(q)) = \xi$ implies $< w, \dot{q} >= 0$, it follows that

$$\sum_{k=1}^{4}\left(w^k\ddot{q}^k + \left(\frac{\mathrm{d}}{\mathrm{d}t}w^k\right)\dot{q}^k\right) = 0 \qquad (32)$$

Substituting this into (31), we obtain

$$\lambda(t) = \frac{1}{w^{\mathrm{T}}G^{-1}w}\left\{\sum_k\left(\dot{w}^k\dot{q}^k - \sum_{i,j}w^k\Gamma^k_{ij}\dot{q}^i\dot{q}^j\right)\right\} \qquad (33)$$

From the Riemannian geometry, the constraint force $\lambda(t)$ with the grad $\{\varphi(\boldsymbol{x}(q))\}$ stands for a component of the image space of w $(= J^{\mathrm{T}}(q)\partial\varphi/\partial\boldsymbol{x})$ that is orthogonal to the kernel TF_ξ of w. In other words this component is canceled out by the image space component of the left hand side of (31). From the physical point of view, $\lambda(t)$ should be regarded as a magnitude of the constraint force that presses the surface $\varphi(\boldsymbol{x}(q)) = \xi$ in its normal direction. In order to compromise the mathematical argument with such physical reality, let us suppose that the actuators can supply the torque signal

$$u = \lambda_d \cdot J^{\mathrm{T}}(q)\frac{\partial\varphi}{\partial\boldsymbol{x}} + g(q) \qquad (34)$$

Then, by substituting this into (26) we obtain the Lagrange equation of motion under the constraint $\varphi(\boldsymbol{x}(q)) = \xi$:

$$\begin{aligned} G(q)\ddot{q} + \left\{\frac{1}{2}\dot{G} + S\right\}\dot{q} &= -(\lambda - \lambda_d)J^{\mathrm{T}}(q)\frac{\partial\varphi}{\partial\boldsymbol{x}} \\ &= -\Delta\lambda J^{\mathrm{T}}(q)\frac{\partial\varphi}{\partial\boldsymbol{x}} \qquad (35) \end{aligned}$$

where $\Delta\lambda = \lambda - \lambda_d$. It should be noted that introduction of the first term of control signal of (34) does not affect the solution orbit on the constraint manifold and further it keeps the constraint condition during motion by rendering $\lambda(t)$ $(= \lambda_d + \Delta\lambda(t))$ positive. In a mathematical sense, exertion of the joint torque $\lambda_d J^{\mathrm{T}}(\partial\varphi/\boldsymbol{x})$ plays a role of "lifting" of the image space spanned from the gradient. Further, note that (35) is of an implicit function form with the Lagrange multiplier $\Delta\lambda$. To affirm the argument of treatment of the geodesics through this implicit form, we show an explicit form of the Lagrange equation expressed on the orthogonally projected space (kernel space) by introducing the orthogonal coordinate transformation

$$\dot{q} = (P, \|w\|^{-1}w)\begin{pmatrix}\dot{\boldsymbol{\eta}} \\ \dot{z}\end{pmatrix} = Q\dot{\bar{q}} \qquad (36)$$

where P is a 4×3 matrix whose column vectors with the unit norm are orthogonal to w and $\dot{\boldsymbol{\eta}}$ denotes a 3×1 matrix (3-dim. vector) and $\dot{z}$ a scalar. Since Q is an orthogonal matrix, $Q^{-1} = Q^{\mathrm{T}}$. Hence, if $\dot{q} \in \ker(w)$ $(= TF_\xi)$ then $\dot{z} = 0$. Restriction of (35) to the kernel space of w can be obtained by multiplying (35) by P^{T} from the left in such a way that

$$P^{\mathrm{T}}G(q)\frac{\mathrm{d}}{\mathrm{d}t}(P\dot{\boldsymbol{\eta}}) + P^{\mathrm{T}}\left\{\frac{1}{2}\dot{G} + S\right\}P\dot{\boldsymbol{\eta}} = 0 \qquad (37)$$

which is reduced to the Euler equation in $\dot{\boldsymbol{\eta}}$:

$$\bar{G}(q)\ddot{\boldsymbol{\eta}} + \left\{\frac{1}{2}\dot{\bar{G}} + \bar{S}\right\}\dot{\boldsymbol{\eta}} = 0 \qquad (38)$$

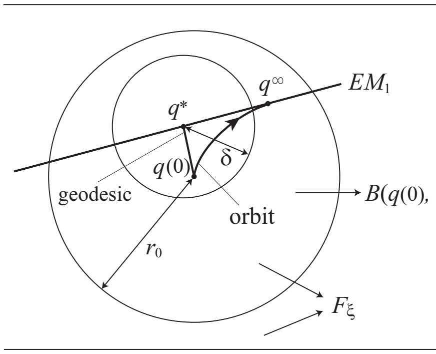

Fig. 5. Definition of the Riemannian ball $B(q^*, \delta)$. Any orbit starting from $B(q^*, \delta)$ converges to $EM_1 \cap B(q^*, \varepsilon)$

or equivalently

$$\ddot{\eta}^k + \bar{\Gamma}^k_{ij}\dot{\eta}^i\dot{\eta}^j = 0, \qquad k = 1, \cdots, n-1 (=3) \tag{39}$$

where $\bar{G}(q) = P^{\mathrm{T}}G(q)P$, $\bar{\Gamma}^k_{ij}$ the Christoffel's symbol for $(\bar{g}_{ij}) = \bar{G}$, and

$$\bar{S} = P^{\mathrm{T}}SP - \frac{1}{2}\dot{P}^{\mathrm{T}}GP + \frac{1}{2}P^{\mathrm{T}}G\dot{P} \tag{40}$$

which is skew-symmetric, too. Note that the transformation Q is isometric and equation (39) stands for the geodesic equation on the constraint Riemannian submanifold.

Let us now reconsider a hybrid position/force feedback control scheme, which is of the form

$$u = g(q) + \lambda_d J^{\mathrm{T}}(q)\frac{\partial\varphi}{\partial \boldsymbol{x}} + C\dot{q} + J^{\mathrm{T}}(q)(\zeta\sqrt{k}\dot{\boldsymbol{x}} + k\Delta\boldsymbol{x}) \tag{41}$$

where $\varphi(\boldsymbol{x}) = z$, $\dot{\boldsymbol{x}} = (\dot{x}, \dot{y}, 0)^{\mathrm{T}}$, and $\Delta\boldsymbol{x} = (x - x_d, y - y_d, 0)$. This type of hybrid control is a modification of McClamroch and Wang's scheme [6], [7] to cope with a robotic system that is subject to redundancy in DOFs. Substituting this into (26) yields

$$G(q)\ddot{q} + \left\{\frac{1}{2}\dot{G} + S + C\right\}\dot{q} + J^{\mathrm{T}}(q)\left\{\zeta\sqrt{k}\dot{\boldsymbol{x}} + k\Delta\boldsymbol{x}\right\} = -\Delta\lambda\frac{\partial z(q)}{\partial q} \tag{42}$$

Evidently the inner product of (42) and $\dot{q}$ under the constraint $z(q) = \xi$ leads to

$$\frac{\mathrm{d}}{\mathrm{d}t}E(q, \dot{q}) = -\dot{q}C\dot{q} - \zeta\sqrt{k}\|\dot{\boldsymbol{x}}\|^2 \tag{43}$$

where

$$E(q, \dot{q}) = \frac{1}{2}\dot{q}^{\mathrm{T}}G(q)\dot{q} + \frac{k}{2}\|\Delta\boldsymbol{x}\|^2 \tag{44}$$

Unfortunately, this quantity is not positive definite in the tangent bundle TF_ξ. Nevertheless, it is possible to see that magnitudes of $\dot{q}$ and $\Delta\boldsymbol{x}$ remain small if, at initial time $t = 0$, $\|\dot{q}(0)\|$ and $\|\Delta\boldsymbol{x}(0)\|$ remain small. Let us introduce the equilibrium manifold as

$$EM_1 = \{q | z(q) = \xi,\ x(q) = x_d,\ y(q) = y_d\} \tag{45}$$

which is of one-dimension. Fortunately as discussed in the paper [8], it is possible to confirm that a modified scalar function

$$W_\alpha = E(q, \dot{q}) + \alpha\dot{q}^{\mathrm{T}}G(q)P_\varphi J^{\mathrm{T}}(q)\Delta\boldsymbol{x} \tag{46}$$

becomes positive definite in TF_ξ with an appropriate positive parameter $\alpha > 0$, provided that the physical scale of the robot is ordinary and feedback gains C and k are chosen adequately. Here in (46) P_φ is defined as $I_4 - ww^{\mathrm{T}}/\|w\|^2$. Furthermore, if in a Riemannian ball in F_ξ defined as $B(q(0), r_0) = \{q | d(q, q(0)) < r_0\}$ the Jacobian matrix $J(q)$ is nondegenerate and there exist positive numbers σ_m and σ_M such that

$$\sigma_M I_2 \geq JJ^{\mathrm{T}} \geq JP_\varphi J^{\mathrm{T}} \geq \sigma_m I_2 \tag{47}$$

then it is possible to show that

$$(1 - \alpha\gamma_0)E \leq W_\alpha \leq (1 + \alpha\gamma_0)E \tag{48}$$

$$\dot{W}_\alpha \leq \frac{-\alpha\sigma_m}{1 + \alpha\gamma_0}W_\alpha \tag{49}$$

where γ_0 can be set as $\gamma_0 = \sigma_M/k$. Hence, by choosing $\alpha = 1/2\gamma_0 = k/2\sigma_M$, it follows from (49) and (48) that

$$W_\alpha(t) = W_\alpha(0)e^{-\gamma t} \tag{50}$$

where $\gamma = k\sigma_m/3\sigma_M$.

Now, suppose that q^* in $EM_1 \cap B(q(0), r_0)$ is the minimizing point that connects with $q(0)$ among all geodesics from $q(0)$ to any point of $EM_1 \cap B(q(0), r_0)$. We call q^* a reference point corresponding to $q(0)$.

Definition (Stable Riemannian Ball on a Submanifold)

If for any $\varepsilon > 0$ there exists a number $\delta(\varepsilon) > 0$ such that any solution trajectory (orbit) of (42) starting from an arbitrary initial position inside $B(q^*, \delta(\varepsilon))$ with $\dot{q}(0) = 0$ remains inside $B(q^*, \varepsilon)$ for any $t > 0$, then the reference point q^* on EM_1 is said to be stable on a submanifold (see Fig. 5).

It can be concluded from the exponential convergence of W_α to zero that any point inside $B(q^*, \delta(\varepsilon))$ included in $B(q(0), r_1)$ for some r_1 ($< r_0$) is stable on a submanifold and further such a solution trajectory converges asymptotically to some q^∞ on the equilibrium manifold EM_1 in an exponential speed of convergence. This can be well understood as a natural extension of the well-known Dirichlet-Lagrange stability under holonomic constraints to a system with DOF-redundancy.

V. 2-DIMENSIONAL STABLE GRASP OF A RIGID OBJECT WITH ARBITRARY SHAPE

Consider a control problem for stable grasping of a 2-D rigid object by a pair of planar multi-joint robot fingers with hemispherical finger-tips as shown in Fig. 6. In this figure, the two robots are installed on the horizontal xy-plane $\boldsymbol{E}^2$. We denote the object mass center by O_m with the coordinates (x_m, y_m) expressed in the inertial frame. On the other hand, we express a local coordinate system fixed at the object by O_m-XY together with unit vectors $\boldsymbol{r}_X$ and $\boldsymbol{r}_Y$ along the X-axis and Y-axis respectively (see Fig. 7). The left-hand side surface of the object is expressed by a curve $c(s)$ with local coordinates $(X(s), Y(s))$ in terms of length parameter s as shown in Fig. 7.

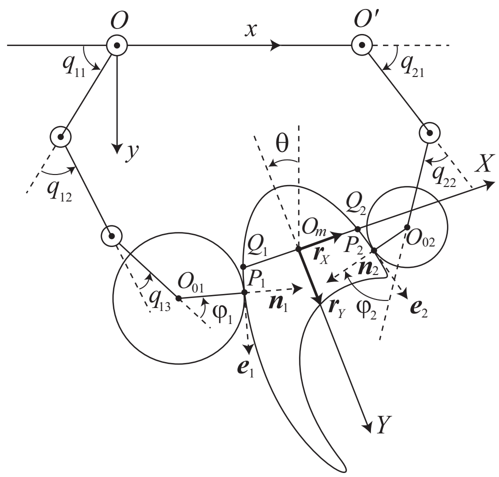

Fig. 6. Two-dimensional object grasping by a pair of multi-joint robot fingers

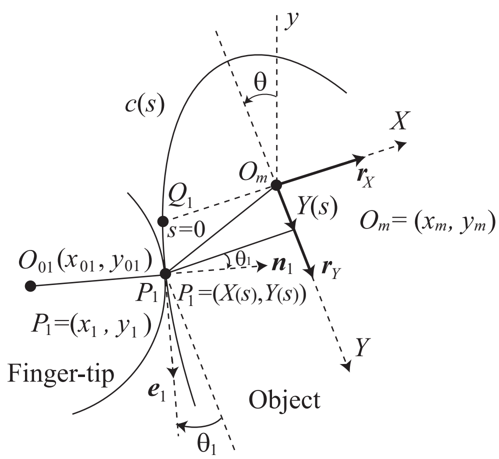

Fig. 7. Definitions of physical symbols in relation to rolling contact. θ_1 denotes $\theta_1(s)$ appearing in (51)

First, suppose that at the left-hand contact point P_1 the finger-tip of the left finger is contacting with the object. Denote the unit normal at P_1 by $\boldsymbol{n}_1$ and the unit tangent vector by $\boldsymbol{e}_1$. Note that $\boldsymbol{n}_1$ is normal to both the object surface and finger-end sphere at P_1 and $\boldsymbol{e}_1$ is tangent to them at P_1, too. If we denote position P_1 by local coordinates $(X(s), Y(s))$ fixed at the object (see Fig. 7), then the angle from the X-axis to the unit normal $\boldsymbol{n}_1$ is assumed to be determined by a function on the curve:

$$\theta_1(s) = \arctan(X'(s)/Y'(s)) \tag{51}$$

where $X'(s) = \mathrm{d}X(s)/\mathrm{d}s$ and $Y' = \mathrm{d}Y(s)/\mathrm{d}s$. Then,

$$\overline{P_1P_1'} = l_{n1}(s) = -X(s)\cos\theta_1(s) + Y(s)\sin\theta_1(s) \tag{52}$$

which is dependent only on s and therefore a shape-function of the object. On the other hand, this arclength can be expressed by using the inertial frame coordinates in the following way (see Fig. 8):

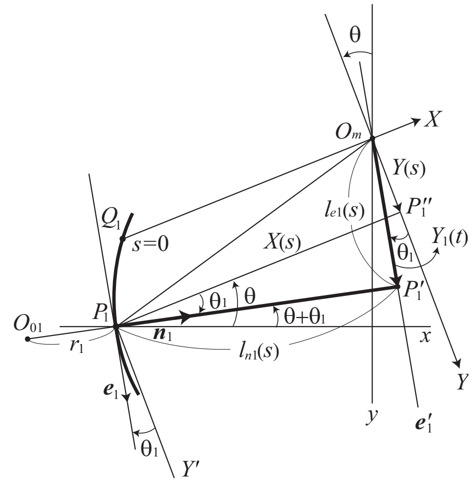

Fig. 8. Physical and geometrical meanings of key variables $l_{n1}(s)$ and $l_{e1}(s)$ defined in relation to the object contour $c(s)$ expressed in terms of length parameter s of the curve

$$\begin{aligned}\overline{P_1P_1'} &= (x_m - x_{01})\cos(\theta + \theta_1(s)) \\ &\quad -(y_m - y_{01})\sin(\theta + \theta_1(s)) - r_1\end{aligned} \tag{53}$$

Hence, the left-hand contact constraint can be expressed by the holonomic constraint

$$\begin{aligned}Q_1 &= -(x_m - x_{01})\cos(\theta + \theta_1) \\ &\quad +(y_m - y_{01})\sin(\theta + \theta_1) + (r_1 + l_{n1}(s)) \\ &= 0\end{aligned} \tag{54}$$

where $l_{n1}(s)$ denotes the right hand side of (52).

Next, we note that the length $\overline{O_mP_1'}$ can also be regarded as a shape-function of the object as in the following:

$$\overline{O_mP_1'} = l_{e1}(s) = X(s)\sin\theta_1 + Y(s)\cos\theta_1 \tag{55}$$

On the other hand, this quantity can be also expressed as

$$\begin{aligned}\overline{O_mP_1'} = Y_1(t) &= -(x_m - x_{01})\sin(\theta + \theta_1) \\ &\quad -(y_m - y_{01})\cos(\theta + \theta_1)\end{aligned} \tag{56}$$

Since the rolling contact shares a common tangent [9], we see that

$$\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{r}_{01} - \boldsymbol{r}_m) = r_1\dot{p}_1\boldsymbol{e}_1 + l_{b1}\dot{\theta}\boldsymbol{n}_1 + l_{n1}\dot{\theta}\boldsymbol{e} \tag{57}$$

where $\boldsymbol{r}_{01}$ and $\boldsymbol{r}_m$ denote the position vectors of O_{01} and O_m in the frame coordinates, $\boldsymbol{e}_1 = (\cos(\theta+\theta_1), -\sin(\theta+\theta_1))^{\mathrm{T}}$, and $\boldsymbol{n}_1 = (\sin(\theta+\theta_1), \cos(\theta+\theta_1))^{\mathrm{T}}$. Taking the inner product of (57) and $\boldsymbol{e}_1$ yields

$$(\dot{\boldsymbol{r}}_{01} - \dot{\boldsymbol{r}}_m)^{\mathrm{T}}\boldsymbol{e}_1 = \sum_{i=1}^{3} r_1\dot{q}_{1i} + l_{n1}\dot{\theta} \tag{58}$$

This is regarded as a Pfaffian constraint of rolling. Further, the arclength parameter should be updated by the first-order differential equations:

$$\frac{\mathrm{d}s_i}{\mathrm{d}t} = \frac{r_i}{1 + r_i\kappa_i(s)}(\dot{p}_i - \dot{\theta}), \quad i = 1, 2 \tag{59}$$

which comes from differention of the equality $\boldsymbol{e}_0(p_1) = \boldsymbol{e}_1(\theta, s)$, that is, the contact point shares the same tangent, where $p_1 = q_{11} + q_{12} + q_{13}$ and $p_2 = q_{21} + q_{22}$. As to the contact point P_2 at the right-hand fingerend, a similar holonomic constraint can be obtained. Thus, by introducing Lagrange's multipliers f_i associated with constraints $Q_i = 0$ $(i = 1, 2)$, it is possible to construct a Lagrangian:

$$L = \frac{1}{2}\sum \dot{q}_i^{\mathrm{T}} G_i(q_i)\dot{q}_i + \frac{1}{2}\left\{M\dot{x}^2 + M\dot{y}^2 + I\dot{\theta}^2\right\} - f_1 Q_1 - f_2 Q_2 \quad (60)$$

where q_i denote the joint vector for finger i, $G_i(q_i)$ the inertia matrix for finger i, M and I denote the mass and inertia moment of the object. Since both the rolling constraints are Pfaffian, it is possible to associate (58) and its corresponding form at P_2 with another multipliers λ_i $(i = 1, 2)$ and regard them as external forces. Thus, by applying the variational principle to the Lagrangian together with the external forces, we obtain the Lagrange equation of motion of the overall fingers-object system:

$$I\ddot{\theta} - f_1 Y_1 + f_2 Y_2 - \lambda_1 l_{n1} + \lambda_2 l_{n2} = 0 \quad (61)$$

$$M\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} - f_1\boldsymbol{n}_1 - f_2\boldsymbol{n}_2 - \lambda_1\boldsymbol{e}_1 - \lambda_2\boldsymbol{e}_2 = 0 \quad (62)$$

$$G_i(q_i)\ddot{q}_i + \left\{\frac{1}{2}\dot{G}_i + S_i\right\}\dot{q}_i + f_i J_i^{\mathrm{T}}(q_i)\boldsymbol{n}_i + \lambda_i\left\{J_i^{\mathrm{T}}(q_i)\boldsymbol{e}_i + (-1)^i r_i \boldsymbol{p}_i\right\} = u_i, \quad i = 1, 2 \quad (63)$$

where $\boldsymbol{p}_1 = (1, 1)^{\mathrm{T}}$, $\boldsymbol{p}_2 = (1, 1, 1)^{\mathrm{T}}$, and $J_i^{\mathrm{T}}(q_i) = \partial(x_{0i}, y_{0i})/\partial q_i$, and

$$\boldsymbol{n}_2 = \begin{pmatrix} -\cos(\theta + \theta_2) \\ \sin(\theta + \theta_2) \end{pmatrix}, \quad \boldsymbol{e}_2 = \begin{pmatrix} \sin(\theta + \theta_2) \\ \cos(\theta + \theta_2) \end{pmatrix} \quad (64)$$

for $i = 1, 2$. The object dynamics of (61) and (62) can be recast in

$$H_0\ddot{\boldsymbol{z}} + f_1\boldsymbol{w}_1 + f_2\boldsymbol{w}_2 + \lambda_1\boldsymbol{w}_3 + \lambda_2\boldsymbol{w}_4 = 0 \quad (65)$$

where $\boldsymbol{z} = (x, y, \theta)$ and

$$\boldsymbol{w}_i = \begin{pmatrix} -\boldsymbol{n}_i \\ (-1)^i Y_i \end{pmatrix}, \quad \boldsymbol{w}_j = \begin{pmatrix} -\boldsymbol{e}_j \\ (-1)^i l_{ni} \end{pmatrix}, \quad \begin{matrix} i = 1, 2 \\ j = 3, 4 \end{matrix} \quad (66)$$

which are of a two-dimensional wrench vector. This implies that if the sum of all wrench vectors converges to zero then the force/torque balance is established. Further, define

$$\begin{aligned} &\boldsymbol{X} = (x, y, \theta, q_1^{\mathrm{T}}, q_2^{\mathrm{T}})^{\mathrm{T}}, \ \boldsymbol{u} = 0, 0, 0, u_1^{\mathrm{T}}, u_2^{\mathrm{T}})^{\mathrm{T}} \\ &G(\boldsymbol{X}) = \mathrm{diag}(M, M, I, G_1(q_1), G_2(q_2)) \\ &S(\boldsymbol{X}, \dot{\boldsymbol{X}}) = \mathrm{diag}(0, 0, 0, S_1, S_2) \end{aligned}$$

Then (61) $\sim$ (63) can be written in the form

$$G(\boldsymbol{X})\ddot{\boldsymbol{X}} + \left\{\frac{1}{2}\dot{G} + S\right\}\dot{\boldsymbol{X}} + \sum_{i=1,2}\left(f_i\frac{\partial Q_i}{\partial \boldsymbol{X}} + \lambda_i\frac{\partial R_i}{\partial \boldsymbol{X}}\right) = \boldsymbol{u} \quad (67)$$

where

$$R_i = r_i\{\theta + \theta_i - \boldsymbol{p}_i^{\mathrm{T}} q_i\} + (\boldsymbol{r}_{01} - \boldsymbol{r}_m)^{\mathrm{T}}\boldsymbol{b}_i, \ i = 1, 2 \quad (68)$$

Comparing (67) with (28), it must be understood that a similar but extended argument of section IV can be applied even for a case of physical interaction with complex holonomic constraints and nonholonomic (but Pfaffian) constraints.

VI. Control Signal for Blind Grasping

From the practical standpoint of designing a control signal for stable grasping, it is important to see that objects to be grasped are changeable but the pair of robot fingers are always the same. To this end, let us propose a family of control signals defined as

$$u_i = -c_i\dot{q}_i + \frac{(-1)^i f_d}{r_1 + r_2} J_i^{\mathrm{T}}(q_i)\begin{pmatrix} x_{01} - x_{02} \\ y_{01} - y_{02} \end{pmatrix} - r_i\hat{N}_i\boldsymbol{p}_i \quad i = 1, 2 \ (69)$$

where c_i denotes a damping factor,

$$\hat{N}_i(t) = \gamma_i^{-1} r_i \boldsymbol{p}_i^{\mathrm{T}}(q_i(t) - q_i(0)) \quad (70)$$

and γ_i a positive gain for $i = 1, 2$. Then, it is possible to show that the closed-loop dynamics is expressed as

$$G(\boldsymbol{X})\ddot{\boldsymbol{X}} + \left\{\frac{1}{2}\dot{G} + S + C\right\}\dot{\boldsymbol{X}} + \frac{\partial P}{\partial \boldsymbol{X}} + \frac{\partial \Phi}{\partial \boldsymbol{X}}\Delta\boldsymbol{\lambda} = 0 \quad (71)$$

where $\Phi = (Q_1, R_1, Q_2, R_2)$, $\Delta\boldsymbol{\lambda} = (\Delta f_1, \Delta\lambda_1, \Delta f_2, \Delta\lambda_2)$, and

$$P = \frac{f_d}{2(r_1 + r_2)}\|\boldsymbol{r}_{01} - \boldsymbol{r}_{02}\|^2 + \sum_{i=1,2}\frac{\gamma}{2}\hat{N}_i^2 \quad (72)$$

We omit in this paper the definitions of f_{id} and λ_{id} in $\Delta f_i = f_i - f_{id}$ and $\Delta\lambda_i = \lambda_i - \lambda_{id}$. It is concluded that any solution $\boldsymbol{X}(t)$ to (71) converges to an equilibrium manifold that minimizes the potential P.

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